UNIVERSITÄT
BERN

Faculty of Economics and
Social Sciences

Department of Economics

What Really Matters: Discounting, Technological Change and Sustainable Climate

Georg Müller-Fürstenberger
Gunter Stephan

10-08

May 2010

DISCUSSION PAPERS

Schanzeneckstrasse 1
Postfach 8573
CH-3001 Bern, Switzerland
http://www.vwi.unibe.ch

What Really Matters: Discounting, Technological Change and Sustainable Climate

Georg Müller-Fürstenberger[a] and Gunter Stephan[b1]

[a]Department of Economics, Universität Trier

[b]Department of Economics and OCCR Climate, Universität Bern

May 2010

Abstract:

This paper discusses the interplay between the choice of the discount rate, greenhouse gas mitigation and endogenous technological change. Neglecting the issue of uncertainty it is shown that the green golden rule stock of atmospheric carbon is uniquely determined, but is not affected by technological change. More general it is shown analytically within the framework of a reduced model of integrated assessment that optimal stationary stocks of atmospheric carbon depend on the choice of the discount rate, but are independent of the stock of technological knowledge. These results are then reinforced numerically in a fully specified integrated assessment analysis.

JEL Classification: Q40, O13.

Key-Words: Integrated Assessment, discount rate, endogenous technological change, climate change.

[1]The authors gratefully acknowledge helpful comments by Raphael Bucher, Jeremy Lucchetti, Oliver Schenker and Ralph Winkler. Of course, the usual disclaimer applies. The second author also acknowledges the support of the Swiss National Center of Competence in Research on Climate (NCCR Climate) funded by the Swiss National Science Foundation (SNF).

1 Introduction

Two aspects are of importance for the future of the global climate and, to a certain extent, for the future of the human society. On the one hand this is the timing of greenhouse gas emissions, and it is the de-carbonization of the economy on the other. The reason is quite obvious. Because of the tremendous inertia of the climate system, the earlier greenhouse gas emissions are abated, and the earlier less carbon intense or even carbon free technologies are innovated, the lower will be the human impact on the global climate, and hence the lower are the costs, future generations have to bear.

Today, more than eighty percent of the world's energy demand is covered by fossil fuels. This explains why energy consumption is the most important source of carbon dioxide (CO_2) emissions. And there is still increasing need for energy, mainly because of the energy hunger of the developing countries. Therefore, high demand for fossil fuels is likely to persist into the future. This is in not in line with the policy recommendations of the Intergovernmental Panel on Climate Change (IPCC, 2001). The IPCC suggests a stabilization of the atmospheric carbon concentration, which, however, requires eliminating global carbon emissions almost completely within the next two centuries. Therefore, answering the question of how to de-carbonize the world economy plays a key role in the solution of the global climate problem.

There is empirical evidence that de-carbonization has already taken place in almost any part of the world (see Nakicenovic, 2002). For example, in the United States the amount of carbon that today is emitted per dollar value added accounts to only ten percent of the amount which was released hundred years ago. But despite of that carbon dioxide emissions have risen over the last two centuries and still continue to rise. How can this be explained? Typically it is argued that because of the so-called rebound effect (see Birol and Keppler, 2000) the rate at which de-carbonization has taken place through the innovation of more energy efficient technologies is significantly smaller than the rate of growth of demand for carbon energy and hence of carbon dioxide emissions. Or to phrase it more frankly, economic growth simply has whipped out the de-carbonizing effect of technological change.

This seemingly suggests that putting technological change into the driver's seat is essential for the de-carbonization of the economy, but we cannot fully trust that technological innovation by itself will guarantee a sufficient increase of energy efficiency on the one hand and a reduction of the carbon intensity on the other. Of course, improving the energy efficiency initially stipulates a reduction of energy consumption and hence of emissions. A lower energy bill, however, implies that relative prices change and real incomes increase. Since relative prices matter, as a main lesson of economic theory tells, at the end of the reaction chain overall energy consumption might rise (see Brännlund et al., 2007). What is needed is getting the prices right, which is nothing else than a modern manifestation of Hicks' (1932) induced innovation hypothesis.

Both, greenhouse gas mitigation and technological innovation can be viewed as investment into the future. Consequently, intertemporal prices and thus the discount rate matters. For example, as Stephan and Müller-Fürstenberger (1998) have shown, there is an inverse relationship between the choice of the discount rate and greenhouse gas abatement. A rapid step up in near term abatement, even above long-run efficiency levels, is observed, if the future generations' welfare is discounted at rate zero. In general, choosing a low discount rate implies that a high weight is put on the welfare of the future generations. Therefore one might expect that the lower is the discount rate the more is invested into both the future climate and the stock of technological knowledge.

However, things turn out being more complex. For example, Goulder and Mathai (2000) have observed that there will be a delay in greenhouse gas mitigation, if improvements in abatement technologies are expected. For, if abatement costs will be reduced by technological change, then it is profitable to abate more, but deferred into the future. Consequently, the choice of the discount rate seems to affect the interaction between greenhouse gas abatement and technological innovation.

This paper discusses the interplay between the choice of the discount rate, greenhouse gas mitigation and endogenous technological change both analytically and numerically.[2] By adopting the concept of a maximal sustainable level of both consumption and environmental quality, which Chichilnisky et al. (1995) call green golden rule, it is shown that the green gol-

[2] In the past a lot has been published on the issue of evaluating costs and benefits of greenhouse gas mitigation, on the conflict between intergenerational equity and intertemporal efficiency, on the issue of discounting for the short-run versus for the long-term, as well as on the related problem of how best to discount the future in the case of uncertainty. (For an overview, see Portney and Weyant, 1999). However, these issues are not in the focus of this paper.

den rule stock of atmospheric carbon is uniquely determined, independent of the stock of technological knowledge.

This result is reinforced in a more general fashion. By introducing the concept of a modified green golden rule it is shown analytically within a reduced version of an integrated assessment model that optimal stationary stocks of atmospheric carbon depend on the choice of the discount rate, but are independent of the technological change. This is also observed numerically within a fully specified integrated assessment model. But the numerical model shows a bit more. In particular it allows analyzing in detail, how economies develop over time and during transition to steady state. Thereby it becomes obvious: (1) over the long-run optimal program converge to stationary states, and (2) the process of approaching a stationary atmospheric stock of carbon dioxide is affected by technological change. In other words: What really matters over the long-run is the choice of the discount rate, but how fast a stationary atmospheric stock of carbon dioxide is reached, depends on technological innovation.

The rest of the paper is organized as follows. Section 2 introduces a simple, analytically treatable model, where technological change results from research and development and affects the productivity of both carbon energy and physical capital. Section 3 establishes the concept of a modified green golden rule and presents the main results of this paper. Section 4 discusses numerical results of an integrated assessment analysis of global climate change, which is based on the theoretical approach developed in Section 2. Section 5 concludes.

2 The modeling framework

Before proceeding to a numerical Integrated Assessment Analysis, let us clarify ideas by using a model, which is simple enough to be solved analytically. Time is taken as discrete and global climate change is viewed as public bad, which negatively affects the world's gross production (WP). In each period $t = 0,1,2,3, \ldots$, the world product (net of climate damages) can be consumed or might be invested into physical capital as well as into a stock of technological knowledge. Inputs into world production are knowledge and physical capital. Furthermore, greenhouse gas emissions, which, if measured in carbon equivalents, are directly governed by inputs of carbon energy, are viewed as inputs into production rather than a joint output.

As was pointed out by Romer (1990), knowledge is different from other inputs such as energy or capital. Technological knowledge is an instruction for mixing together energy, raw materials and other services. That means in particular that technological progress introduces new devices into production, which, once discovered, can be applied as often as desired without any restrictions. Therefore, if we allow for greenhouse gas abatement through induced technological change and / or substitution between physical capital and carbon energy, i.e. carbon emissions, the most simply way to represent world production is to use the following function (for a discussion, see Löschel, 2002)

(2.1) $$f(H_t, K_t, e_t) = H_t^{\beta} K_t^{\alpha} e_t^{1-\alpha}.$$

K_t and e_t denote the inputs of conventional capital and carbon emissions, respectively. H_t is stock of technological knowledge and $\alpha, \beta \in (0,1)$ are technology parameters.[3]

How greenhouse gas emissions affect the future climate, is captured by using a Nordhaus representation of the global carbon cycle

(2.2) $$Q_{t+1} = \gamma Q_t + e_t.$$

That means that the future stock Q_{t+1} of atmospheric greenhouse gases depends on the present concentration Q_t as well as global emissions e_t. γ is the factor by which natural abatement reduces existing stocks of atmospheric greenhouse gases.

Let the economic impact of global climate change be measured in terms of losses in world production. That means, global climate change directly affects the regions' ability to produce private goods, but not utilities. Then feasibility requires that

(2.3) $$\theta(Q_t) f(H_t, K_t, e_t) \geq H_{t+1} - \varepsilon_H H_t + K_{t+1} - \varepsilon_K K_t + c_t$$

for any *t*.

The right-hand side of inequality (2.3) indicates that the world product - net of climate damages - is allocated between consumption c_t, investment into physical capital, $K_{t+1} - \varepsilon_K K_t$,

[3] Since knowledge is a factor of production, investing into knowledge capital raises the productivity of resources and result in non-environmental technical progress. As long as the output elasticity of knowledge is positive, the production will be characterized by increasing returns to scale, i.e. by endogenous technological change as referred to by the new growth theory.

where ε_K is the capital survival factor, and investment, $H_{t+1} - \varepsilon_H H_t$, into the knowledge stock, where knowledge will erode at rate $1-\varepsilon_H$ without activity.

The left-hand side of inequality (2.3) denotes the fraction of conventional world output (WP) that is at the society's disposal. This is called Green World Product (GWP) and depends on the economic losses $\theta(Q_t)$, which are negatively correlated to the stock of atmospheric carbon. That means, the more carbon is emitted, the higher will be the stock Q_t of accumulated global emissions, and hence, the lower will be the fraction $\theta(Q_t)f(H_t, K_t, e_t)$ of conventional output that is available in period t. More precisely we assume in the following that damages are strictly increasing, i.e., $\theta'(Q_t) < 0$and $\theta''(Q_t) < 0$ for all t.

An example of an economic loss factor, which typically is used in Integrated Assessment Analysis, where for the sake of simplicity both the thermal inertia lag between global concentrations and climate change as well as the cooling effects of aerosols and the heating effects of greenhouse gases other than carbon dioxide are neglected, is given by (see Manne et al., 1995)

(2.4) $$\theta(Q_t) = 1 - \left(\frac{Q_t}{\Omega}\right)^2.$$

Ω marks the critical value of the CO_2-stock. At this hypothetical level, climate damage would consume all of conventional wealth.[4]

3 Sustainable climate and (Modified) Green Golden Rule

Among climate scientists there is general agreement that over the long run the stock of atmospheric carbon should be stabilized at levels, which are at least below catastrophic ones. Economists typically add that stabilizing the global climate must go hand in hand with an indefinitely maintainable level of consumption per capita. Both requirements are captured by the concept of the green golden rule. This concept was originally introduced by Chichilnisky et al. (1995), and considers a green golden rule a feasible path such that the long-run values of both consumption and the environment are maximized.

[4] In most studies the world's critical CO_2 concentration level Ω is 1496 ppm. This implies that doubling the concentration of pre-industrial atmospheric carbon imposes market losses of 3.5 % of the world's gross product.

To adopt their concept, let U denote instantaneous utility, which is a concave function of consumption c_t. Recall that an infinite sequence $\{H_t, K_t, Q_t, t = 0,1,2,3, \ldots\}$ of stocks of technical knowledge H_t, physical capital K_t and atmospheric carbon Q_t, respectively, is feasible path, if for any t condition (2.3) is fulfilled. A Green Golden Rule (GGR) is a feasible path, which maximizes

$$\lim_{t\to\infty} U(\,\theta(Q_t)f(H_t, K_t, e_t) - (H_{t+1} - \varepsilon_H H_t) - (K_{t+1} - \varepsilon_K K_t)).$$

Proposition 1: Let $\varepsilon_H < 1, \varepsilon_k < 1$, then a Green Golden Rule exists and the GGR stock Q^* of atmospheric carbon satisfies $Q^* = -(1-\alpha)\,\theta(Q^*)/\theta'(\,Q^*)$.

Proof: Since for any t the maximand is independent of the values of H_t, K_t, Q_t, solving the optimality problem requires to find infinitely maintainable values of H, K, Q, which grant maximal consumption. Thus, the above problem reduces to

$$max\, U(\,\theta(Q)H^\beta K^\alpha((1-\gamma)Q)^{1-\alpha} - (1-\varepsilon_H)H - (1-\varepsilon_K)K.$$

Therefore, an interior solution must fulfill

$$\beta\theta(Q)H^{\beta-1}K^\alpha((1-\gamma)Q)^{1-\alpha} - (1-\varepsilon_H) = 0,$$

$$\alpha\theta(Q)H^\beta K^{\alpha-1}((1-\gamma)Q)^{1-\alpha} - (1-\varepsilon_K) = 0,$$

$$\theta'(Q)H^\beta K^\alpha((1-\gamma)Q)^{1-\alpha} + (1-\gamma)(1-\alpha)\theta(Q)H^\beta K^\alpha((1-\gamma)Q)^{-\alpha} = 0.$$

Now, let H^*, K^*, Q^* denote the solution of the above problem. Then condition (3.3) immediately implies

$$\theta'(Q^*)Q^* + (1-\alpha)\theta(Q^*) = 0.$$

This means in particular that, because of the properties of the damage function θ, the atmospheric GGR stock Q^* is uniquely determined and independent of both of the capital and knowledge stock. Moreover, if $\varepsilon_{H,}, \varepsilon_k < 1$, which is a realistic assumption, from the first two conditions the Green Golden Rule stocks of physical capital and knowledge can be derived, which are in turn uniquely determined. ■

Proposition 1 has an important implication. Sustainable climate, which yields maximal per capita consumption over the long-run, is independent of the stock of technological knowledge. Instead the atmospheric Green Golden Rule stock Q^* solely depends on damages as well as the output elasticity α of physical capital. In case of the well-known damage function (2.4) we get

$$Q^* = \sqrt{\frac{1-\alpha}{3-\alpha}}\,\Omega.$$

For an illustrative example let $\alpha = 0.5$. Then the Green Golden Rule carbon concentration would be around 669 ppm. This fits surprisingly well the results of more complex and detailed integrated assessment analyses of global climate change (for example, see Nordhaus and Boyer, 2000). Note, this result implies in particular that the sustainable concentration of atmospheric carbon is above the presently existing level. Or to phrase it differently, compared to optimal sustainable concentrations of atmospheric carbon the present generation enjoys an over-endowment of climate capital. We will return to this issue in Section 4.

The Green Golden Rule is a particular type of a stationary path, i.e. a feasible path $\{H_t, K_t, Q_t, t = 0,1,2,3, \dots\}$ such that $H_t = H, K_t = K, Q_t = Q, c_t = c$ for a t. To see that, let $p_t = U'(c)$ for all t. Then a Green Golden Rule myopically maximizes profits at these prices (for a clarification, see Appendix 1). Now, since the Green Golden Rule is a stationary path which is myopically maximized profits at constant prices, i.e., prices embodying a proportionality factor $\delta = 1$, this motivates to generalize our consideration by introducing the concept of a Modified Green Golden Rule (MGGR) as follows.

Definition: A stationary path, which myopically maximized profits at proportional prices $p_t = \delta^{-t}U(c)$ is called a Modified Green Golden Rule.

Now, since a Modified Green Golden Rule is a feasible growth path, which for all t has to satisfy the stationary conditions $H_t = H, K_t = K, Q_t = Q, c_t = c$, where

$$c = \theta(Q)H^{\beta}K^{\alpha}((1-\gamma)Q)^{1-\alpha} - (1-\varepsilon_H)H - (1-\varepsilon_K)K,$$

then the first order conditions, which are both sufficient and necessary for short-run profit maximization, turn into

(3.1) $$\beta\theta(Q)H^{\beta-1}K^{\alpha}((1-\gamma)Q)^{1-\alpha} + \varepsilon_H - \delta = 0,$$

(3.2) $$\alpha\theta(Q)H^{\beta}K^{\alpha-1}((1-\gamma)Q)^{1-\alpha} + \varepsilon_K - \delta = 0,$$

(3.3) $$\theta'(Q)H^{\beta}K^{\alpha}((1-\gamma)Q)^{1-\alpha} + (\delta-\gamma)(1-\alpha)\theta(Q)H^{\beta}K^{\alpha}((1-\gamma)Q)^{-\alpha} = 0.$$

Therefore, a feasible, stationary program $\{H, K, Q\}$ exists and defines a MGGR, if and only if it is a solution of conditions (3.1a) to (3.3a).

Proposition 2: For any discount factor $\delta > 1$ a uniquely determined Modified Green Golden rule $(H^{\delta}, K^{\delta}, Q^{\delta})$ exists, where the MGGR stock Q^{δ} of atmospheric carbon is independent of both the knowledge stock H^{δ} and the capital stock K^{δ}. Furthermore

$$Q^{\delta} = (\alpha - 1)\frac{\delta-\gamma}{1-\gamma}\frac{\theta(Q^{\delta})}{\theta'(Q^{\delta})}.$$

Proof: From condition (3.3) we get

(3.3a) $$\theta'(Q)(1-\gamma)Q + (\delta-\gamma)(1-\alpha)\theta(Q) = 0,$$

hence

$$\theta'(Q)Q + \frac{(\delta-\gamma)}{(1-\gamma)}(1-\alpha)\theta(Q) = 0.$$

Because of the properties of the damage function θ, this uniquely determines a MGGR stock Q^{δ}, which is independent of the knowledge stock as well as the stock of physical capital. Furthermore, through inserting Q^{δ} into conditions (3.1) and (3.2) both MGGR knowledge and capital stocks can be derived and are unique. ■

Note, condition (3.3) implies

$$-\theta'(Q)\frac{Q}{\theta(Q)} = (1-\alpha)\frac{(\delta-\gamma)}{(1-\gamma)},$$

which means in particular that at a Modified Green Golden Rule the elasticity of the climate damages directly corresponds to the difference of the discount factor and the natural recreating rate. Now, if we a apply the specific damage functions as given in equation (2.4) we get

$$Q^{\delta} = \sqrt{\frac{(1-\alpha)(\delta-\gamma)}{2+\delta+\alpha\gamma-3\gamma-\alpha\delta}}\Omega.$$

For illustrative purposes, let as assume that utilities are discounted at 3%, that $\gamma = 0.99$ and $\alpha = 0.5$. This would imply a stationary stock of atmospheric carbon Q^{δ} of about 980 ppm.

Condition (3.1) implicitly defines the stationary knowledge stock H^{δ} as function of the stock of atmospheric carbon Q^{δ}, the discount factor δ, and the capital stock K^{δ}, i.e. $H^{\delta} = h(Q^{\delta}, \delta, K^{\delta})$. Therefore, by applying the calculus of implicit functions, we have

(3.4) $$\frac{\partial H^{\delta}}{\partial Q^{\delta}} = -\frac{\beta H^{\beta-1}K^{\alpha}((1-\gamma)Q)^{-\alpha}[\theta'(Q)(1-\gamma)Q+(1-\alpha)(1-\gamma)\theta(Q)]}{(\beta-1)\beta\theta(Q)H^{\beta-2}K^{\alpha}((1-\gamma)Q)^{1-\alpha}} = \frac{H[\theta'(Q)Q+(1-\alpha)\theta(Q)]}{(1-\beta)\theta(Q)Q} < 0.$$

The negative sign follows from the assumption that $\beta < 1$, and $\delta > 1 > \gamma$, hence $\frac{(\delta-\gamma)}{(1-\gamma)}$. Thus condition (3.3) implies $\theta'(Q)Q + (1-\alpha)\theta(Q) < 0$. Furthermore we have

(3.5) $$\frac{\partial H^{\delta}}{\partial \delta} = \frac{1}{(\beta-1)\beta\theta(Q)H^{\beta-2}K^{\alpha}((1-\gamma)Q)^{1-\alpha}} < 0,$$

(3.6) $$\frac{\partial H^{\delta}}{\partial K^{\delta}} = -\frac{\alpha\beta\theta(Q)H^{\beta-1}K^{\alpha-1}((1-\gamma)Q)^{1-\alpha}}{(\beta-1)\beta\theta(Q)H^{\beta-2}K^{\alpha}((1-\gamma)Q)^{1-\alpha}} = \frac{\alpha}{(1-\beta)}\frac{H}{K} > 0,$$

Similar conditions hold true, if we replace the knowledge stock by the physical capital stock, i.e., $\frac{\partial K^{\delta}}{\partial \delta} < 0$, $\frac{\partial K^{\delta}}{\partial Q^{\delta}} < 0$, $\frac{\partial K^{\delta}}{\partial H^{\delta}} > 0$.

Proposition 2 and Condition (3.3a) allow defining the stationary stock $Q^{\delta}$of atmospheric carbon as a function of the discount factor δ only. Hence

(3.7) $$\frac{dQ^{\delta}}{d\delta} = -\frac{(1-\alpha)\theta(Q)}{\theta''(Q)(1-\gamma)Q+\theta'(Q)[1-\gamma+(\delta-\gamma)(1-\alpha)]} > 0,$$

which immediately follows from the properties of the damage function θ.

Now, taking together the results observed so far, we are able to analyze how the choice of the discount factor affects the stationary knowledge stock as well as stock of atmospheric carbon. Recall that $H^{\delta}=h(Q^{\delta},\delta,K^{\delta})$ and $K^{\delta}=k(Q^{\delta},\delta,H^{\delta})$. Then because of conditions (3.5) to (3.7), taking the total differentials gives

(3.8) $$\frac{dH^{\delta}}{d\delta}=\left\{\frac{\partial H^{\delta}}{\partial\delta}+\left[\frac{\partial H^{\delta}}{\partial Q^{\delta}}+\frac{\partial H^{\delta}}{\partial K^{\delta}}\frac{\partial K^{\delta}}{\partial Q^{\delta}}\right]\frac{dQ^{\delta}}{d\delta}+\frac{\partial H^{\delta}}{\partial K^{\delta}}\frac{\partial K^{\delta}}{\partial\delta}\right\}\left[1-\frac{\partial H^{\delta}}{\partial K^{\delta}}\frac{\partial K^{\delta}}{\partial H^{\delta}}\right]^{-1}.$$

Because of the consideration above, it follows that the right hands side of equation (2.12) is negative. Furthermore, since

$$\frac{\partial H^{\delta}}{\partial K^{\delta}}\frac{\partial K^{\delta}}{\partial H^{\delta}}=\frac{\alpha}{(1-\beta)}\frac{\beta}{(1-\alpha)}<1,$$

if $\alpha+\beta<1$, we have

Proposition 3: For any $\delta>1$, there exists a uniquely determined Modified Green Golden Rule $(H^{\delta},K^{\delta},Q^{\delta})$. The MGG Rule stock Q^{δ} of atmospheric greenhouse gases depends monotonously increasing on the discount factor δ and if $\alpha+\beta<1$, then the MGGR knowledge stock H^{δ} is negatively related with both Q^{δ} and δ.

With a one-sector growth model a decrease of the discount rate always implies an increase of the optimal steady state capital stock. In the literature this phenomenon is called capital deepening (see Burmeister and Turnovsky, 1972). However, in models with heterogeneous capital such a response cannot be expected in general. For example, Burmeister and Turnovsky have shown that even in a well-behaved Cobb-Douglas world there is no unambiguous capital deepening. The possibility of substitution between different capital goods prohibits any hope that the stocks of different capital goods will – in any case - move into the same direction. Our analysis, however, reveals that there will be no paradoxical effects. In any case there will be capital deepening in the sense that any increase of the discount rate implies a reduction of the stationary stocks of physical capital, knowledge capital as well as environmental capital, i.e., an increase of the stock of atmospheric carbon.

Note finally that even if stationary stocks of atmospheric carbon are independent of technological knowledge, the path at which an economy approaches stationary values might be very well be governed by technological change. This becomes more obvious, once we turn to a dynamic integrated assessment analysis.

4 Computational Experiments

Just at the beginning let us stress the purely illustrative purpose of the following experiments. Or in other words: We are not interested in deriving policy recommendations. We are interested in generating additional insight through numerical thought experiments. As such our computational exercise employs the theoretical model, which we presented in Section2. Compared to other models used in numerical Integrated Assessment Analyses our theoretical approach exhibits a higher degree of abstraction. In particular, inputs of carbon energy into production are not explicitly modeled. Instead, carbon emissions are viewed as inputs into production. This is a new and strong assumption to the Integrated Assessment literature, and it limits the comparability of our results to the results typically derived from models like DICE (see Nordhaus, 1993). Nonetheless, some key elements of our approach correspond one-to-one to modeling blocks of seminal Integrated Assessment Models. Examples are the damage assessment (see (2.4)) on the one hand and the representation of the carbon-cycle on the other (see (2.2)).

Our calculations are based on the assumption that the world economy follows a Ramsey path, striking an optimal balance between consumption and investment into physical as well as knowledge capital. Formally that means solving the optimality problem

$$max \sum_{t=0}^{\infty} \delta^{-t} ln(c_t),$$

subject to

(2.3a) $$c_t = \left\{1 - \left(\frac{Q_t}{\Omega}\right)^2\right\} H_t^{\beta-1} K_t^{\alpha} (Q_{t+1} - \gamma Q_t)^{1-\alpha} - H_{t+1} + \varepsilon_H H_t - K_{t+1} + \varepsilon_K.$$

Solutions are obtained via non-linear programming. Computations are carried out with GAMS/Conopt3. We simulate 350 periods, but show only 250 to hide end of the horizon effects. Additionally, the results have been checked against the numerical solution of the first order optimality conditions.

4.1 Calibration and data

There are only few parameters to be calibrated. Let us assume that output elasticizes of production (see (2.1)) are $\alpha = 0.5$ and $\beta = 0.2$, respectively. The natural rate of carbon decay is taken from the literature and account to one per cent, i.e., $\gamma = 0.99$. Emissions are partially absorbed by the biosphere or stored in the upper ocean, therefore the fraction of annual carbon emissions which eventually enters the atmosphere, is *0.302*. I.e., the fully specified carbon accumulation equation (2.2) reads

$$Q_{t+1} = 0.99Q_t + 0.302e_t.$$

Just as in the existing literature (see Nordhaus and Boyer, 2000) the economic loss function (2.4) is calibrated such a doubling of the pre-industrial carbon concentration causes market damages of 3.5 per cent of world gross output (recall Footnote 4). Depreciation rates on physical capital and on knowledge are $\varepsilon_K = 0.05$ and $\varepsilon_H = 0.29$, respectively (see Bernstein and Manumeas, 2006 for estimates of ε_H).

4.2 Descriptive versus prescriptive view

There are polar views on the issue of global climate change. On one hand, one could take a descriptive view. This would mean to place the global climate problem into the framework of a decentralized market economy and to use the market rate of interest for evaluating both conventional and environmental capital formation. Alternatively, one could take prescriptive approach. This implies that the greenhouse issue is related to the ideas of equity and intergenerational fairness and high weights are put on the welfare of future generations. Since the present paper uses a Ramsey approach, where optimal climate policies are derived through maximizing the sum of the discounted logarithm of consumption, the differences between a descriptive and prescriptive approach manifest through the choice of the utility discount rate. If a prescriptive view is taken welfare is discounted at a rate of 0.5%, whereas a 3% discount rate applies, if a descriptive approach is chosen.

Figures 1 – 4 report the results of our counterfactual analysis. All figures show that over the long-run there is convergence to stationary states. Figure 1 illustrates, the long-run atmospheric carbon stock inversely depends on the choice of the discount rate. This is perfectly in line with what we expect from our theoretical analysis (see Section 3).

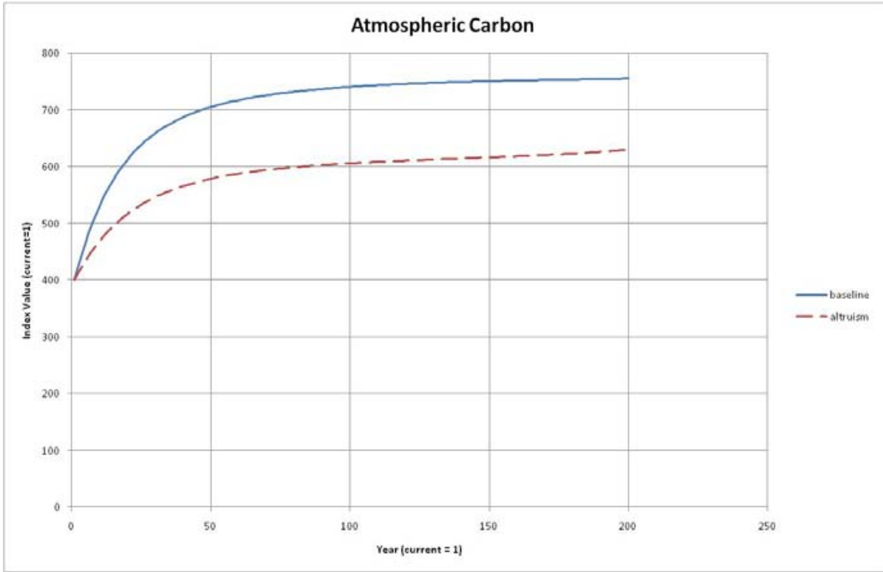

Figure 1: Atmospheric carbon concentration for high (descriptive) and low (prescriptive) discounting.

Figure 2 shows hump-shaped consumption paths. Primarily this results from a high initial endowment in environmental capital, i.e., carbon stocks below stationary stocks, from which present generations can profit as was already mentioned in Section 3. To understand this, recall that our model discriminates between three types of capital: physical capital, knowledge capital and environmental capital, which is the higher the lower is the stock of atmospheric carbon. The initial endowment of both physical and knowledge capital are below their stationary levels, but environmental capital initially is higher than what is optimal over the long-term, i.e., the initial concentration of atmospheric carbon is below (modified) golden rule levels. Consequently current generations can burn large amounts of fossil fuels to bring up the atmospheric carbon stock to the optimal stationary level. Note also, that if a prescriptive view is taken, future generations are favored at the expense of current ones in terms of per-capita consumption, an observation which was already reported in Stephan and Müller-Fürstenberger (1998).

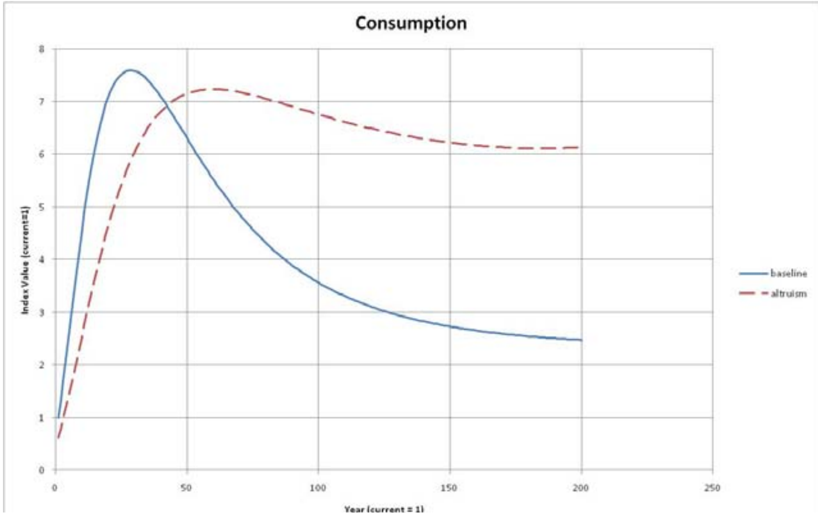

Figure 2: Consumption is hump shaped

Figure 3 and Figure 4 show that physical capital and knowledge follow a similar pattern. This is to be expected as they differ only with respect to output elasticity and depreciation rate. Both stocks overshoot. Again this is due to the initial over-endowment in environmental capital, which allows present generation to extend green world product over stationary levels and hence to invest and consume extensively.

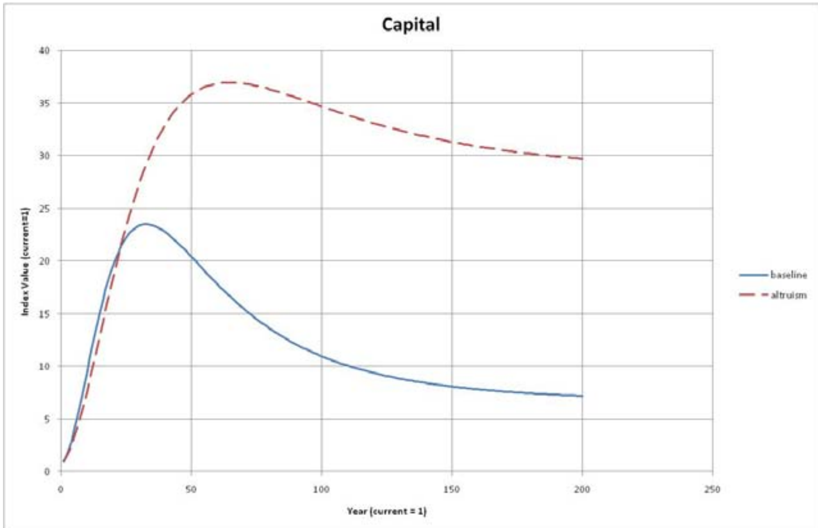

Figure 3: Physical capital

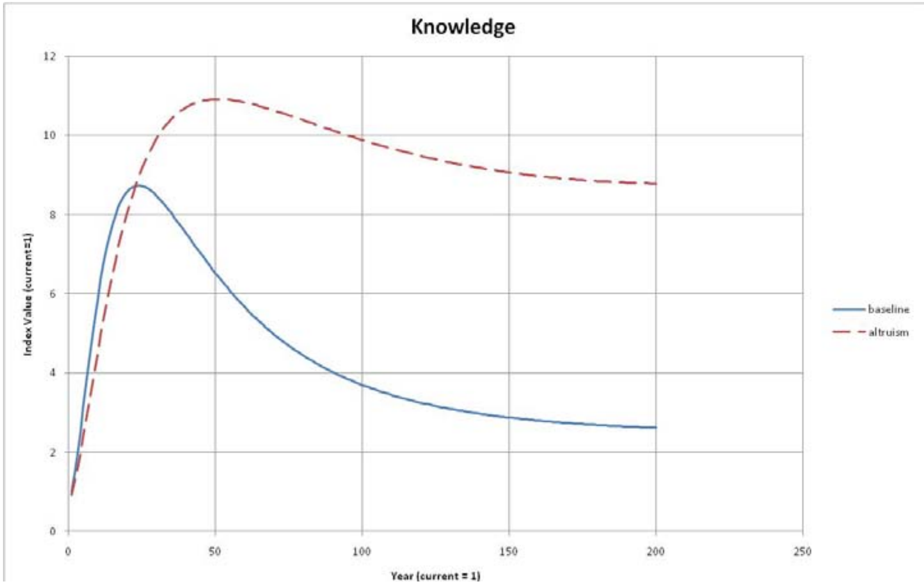

Figure 4: Knowledge capital

Furthermore, Figures 1-4 show that there is capital deepening in any stock, just as was to be expected from the theoretical analysis in Section 3.

4.3 Invariance result

Section 3 has established an invariance result in the sense that the stationary stock Q^{δ} of atmospheric carbon neither depends on the knowledge stock H^{δ} nor on the capital stock K^{δ} (see Propositions 1 and 2). One way to investigate this result numerically is to vary the productivity of technological knowledge, i.e., the output elasticity β of knowledge (see condition (2.1)). Ceteris paribus this implies higher investment into the stock of in production and hence, higher stocks over the long run. This is exactly what Figure 5 illustrates in case of three different values of β (0.2 (baseline), 0.3 (beta_3), 0.35 (beta_35).

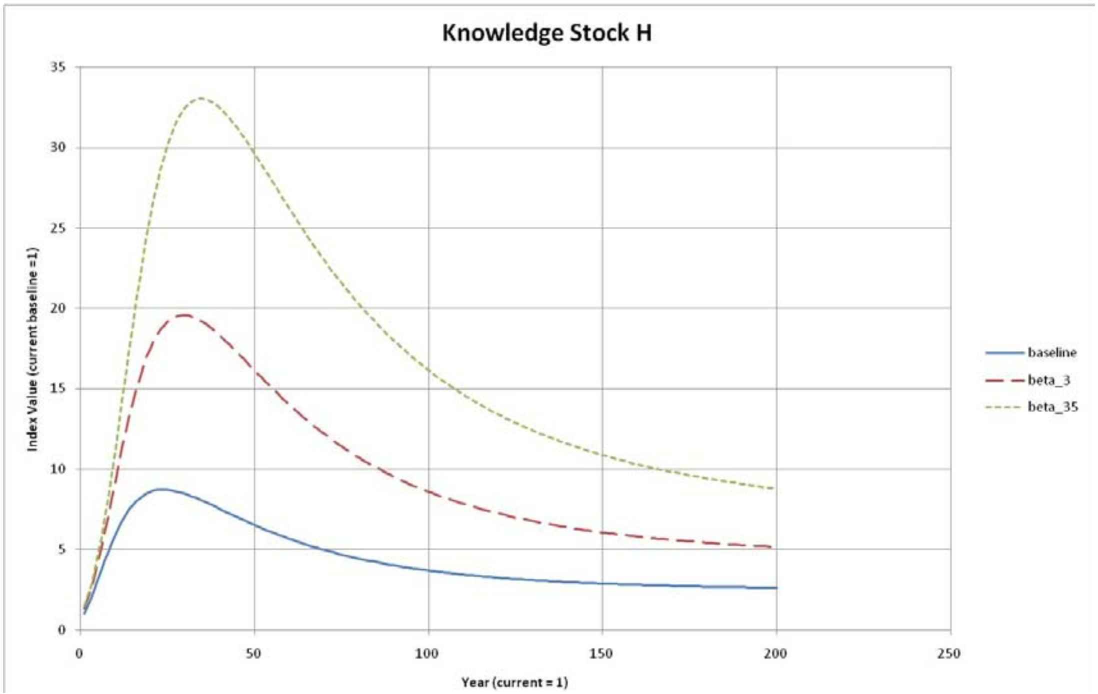

Figure 5: Knowledge stocks for β = 0.2 (baseline), 0.3 (beta_3), 0.35 (beta_35)

The invariance result, which is stated in Proposition 2, is shown in Figure 6. More precisely, Figure 6 demonstrates that over the long-run there is convergence to a sustainable stock of atmospheric carbon, which is independent of the stock of technological knowledge. However, as is also obvious from Figure 6 this does not apply in the transition phase. High output elasticity of knowledge supports slightly higher atmospheric carbon stocks during transition. We call this phenomenon a *transitory rebounding effect*.

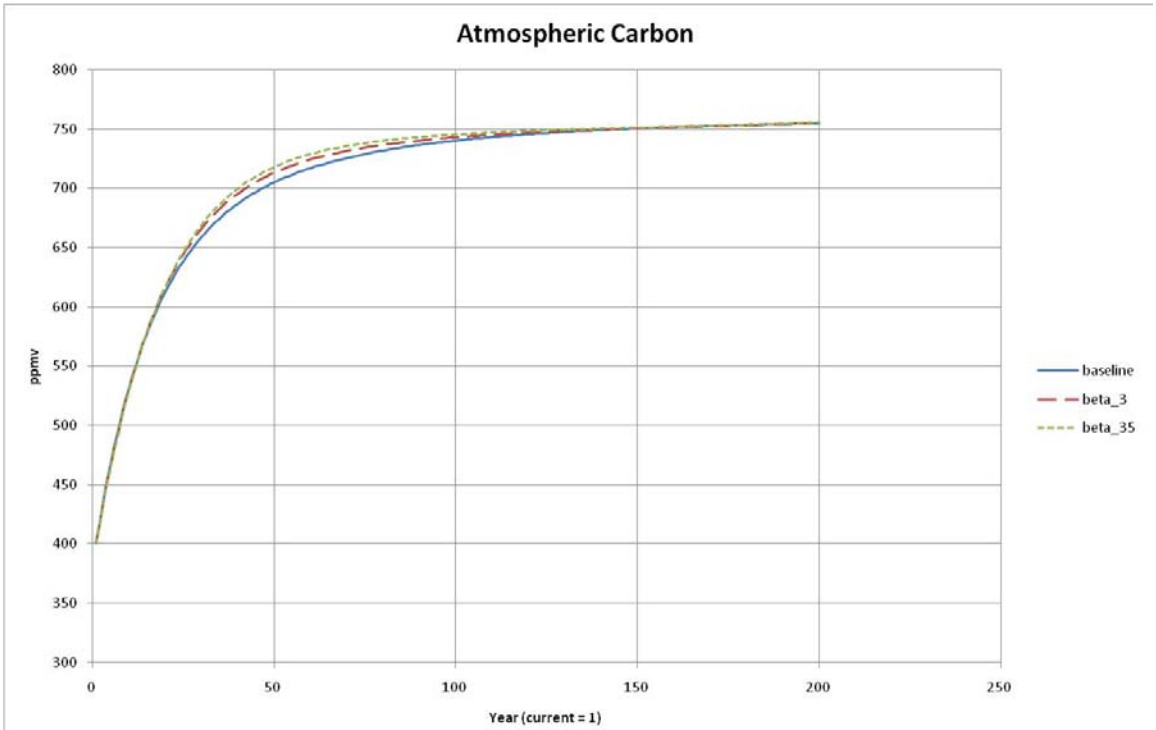

Figure 6: Atmospheric Carbon for β = 0.2 (baseline), 0.3 (beta_3), 0.35 (beta_35)

Figure 7 indicates small long term differences in consumption. However, there are huge differences during transition in absolute terms. With a focus on steady states only, consumption is weakly sensitive on the output elasticity of knowledge only. During transition, technological progress enhances consumption significantly for a long period of time.

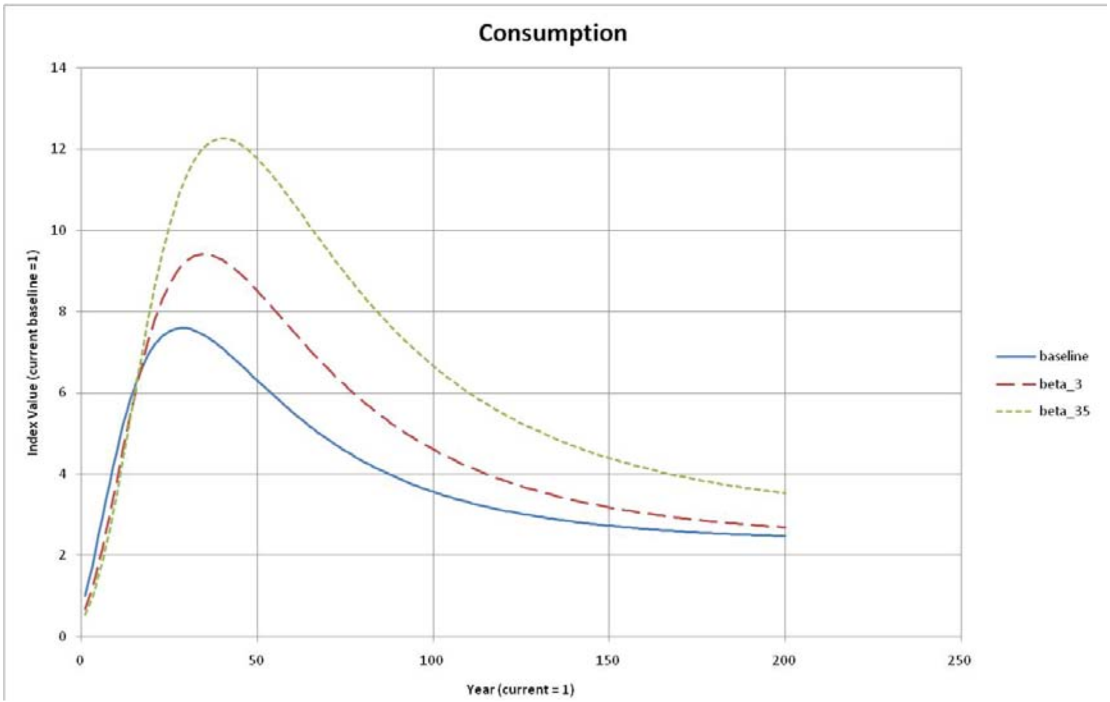

Figure 7: Consumption for β = 0.1 (low_beta), 0.2 (baseline), 0.3 (high_beta).

5 Conclusions

This paper gives new and surprising insight into the relation between technological change and climate policy. By means of a highly stylized integrated assessment model we have shown both theoretically and numerically that the stock of technological knowledge has no impact on what the optimal carbon stock is over the long run. This drastically simplifies negotiations on a climate treaty because policy makers need not to agree on expectations about future technological change. It simply does not matter for a global long run emission target. We are aware that our result does not hold for a more detailed modelling of the energy sector. But at the very bottom line there is an invariance result.

Our analysis emphasises once more the key role of the discount rate. Low discount rates support strong carbon targets, but at the cost of significant consumption losses for current generations. Assuming that there is no exogenous technological change and no population growth, our simulations reveal a hump-shaped pattern of consumption. The hump is more pronounced for higher discount rates than for lower ones. Given that current carbon stocks are below the long-run stationary stocks, there are generations which can heavily benefit from exploiting this gap. Even for low discount rates, future generations may not experience the same welfare as do less distant ones.

6 References

Bernstein, J.I. and T.P. Manumeas (2006). "R&D depreciation, stocks, user costs and productivity growth for US R&D intensive industries. *Structural Change and Economic Dynamics* 17, 70-89.

Birol, F., Keppler, J.H. (2000): "Prices, technology Development and the Rebound Effect". *Energy Policy* 28:457-469.

Brännlund, R., Ghalwash,T., Nordström, J. (2007): "Increased Energy Efficiency and the Rebound Effect: Effects on Consumption and Emissions". *Energy Economics* 29:1-17.

Burmeister, E., Turnovsky, S.J. (1972): "Capital Deepening Response in an Economy with Heterogeneous Capital Goods". *American Economic Review* 62:842-853.

Chichilnisky, G., Heal, G., Beltratti, A. (1995): "The Green Golden Rule". *Economic Letters* 49:175-179.

Goulder, L.H., Mathai, K. (2000): "Optimal CO2 Abatement in the Presence of Induced Technological Change". *Journal of Environmental Economics and Management* 39:1-38.

Hicks, J. (1932), *The Theory of Wages*. Macmillian, London.

IPCC (2001), *Climate Change 2001: Mitigation. Contributions of the Working Group III on the Third Assessment Report of the Intergovernmental Panel on Climate Change*. Cambridge University Press, Cambridge UK.

Löschel, A. (2002), "Technological Change in Economic Models of Environmental Policy: a Survey". *Ecological Economics 43:105- 126.*

Manne, A., Mendelsohn, R., Richels R. (1995), "MERGE: a model for evaluating regional and global effects of GHG reduction policies". *Energy Policy* 3:1.

Mas-Colell, A., Winston, M.D., Green, J.R. (1995), *Microeconomic Theory*. Oxford University Press, New York.

Nakicenovic, N. (2002), "Technological Change and Diffusion as a Learning Process". In *Technological Change and the Environment*. Resources for the Future, Washington, 160-181.

Nordhaus, W.D (1993), "Rolling the 'DICE': An optimal transition path for controlling greenhouse gases." *Resource and Energy Economics* 15:27-50

Nordhaus, W. and Boyer, J. (2000), *Warming the World*. MIT Press, Cambridge.

Portney, P., Weyant, J., (eds.) (1999): *Discounting and Intergenerational Equity*. Resources for the Future, Washington.

Romer, P.M. (1990): "Endogenous Technological Change". *Journal of Political Economy* 98:71-102.

Stephan, G. (1995), *Introduction into Capital Theory*. Springer, Heidelberg.

Stephan, G., Müller-Fürstenberger, G. (1998), "Discounting and the Economic Costs of Altruism in Greenhouse Gas Abatement". *Kyklos* 51:321-338.

Appendix 1:

A1 Myopic profit maximization

Let $\{H_t, K_t, Q_t, t = 0,1,2,3, \dots\}$ be a feasible program. At beginning of period t inputs into production are: H_t, K_t, Q_t. At the of period t outputs are: $\theta(Q_t)f(H_t, K_t, Q_{t+1} - \gamma Q_t)$, $\varepsilon_H H_t$, $\varepsilon_K K_t$, Q_{t+1}. Now let $p_t \geq 0$ and $w_t \leq 0$ denote the present value prices of produced commodities and carbon stocks, respectively[5]. Then short-run or myopic profit function is

$$w_t Q_{t+1} + p_t(\theta(Q_t)f(H_t, K_t, Q_{t+1} - \gamma Q_t) + \varepsilon_H H_t + \varepsilon_K K_t) - p_{t-1}(H_t + K_t) - w_{t-1}Q_t.$$

Therefore solving the myopic maximization problem gives

(A1) $p_t\{\beta\theta(Q_t)H_t^{\beta-1}K_t^{\alpha}(Q_{t+1} - \gamma Q_t)^{1-\alpha} + \varepsilon_H\} = p_{t-1},$

(A2) $p_t\{\alpha\theta(Q_t)H_t^{\beta}K_t^{\alpha-1}(Q_{t+1} - \gamma Q_t)^{1-\alpha} + \varepsilon_K\} = p_{t-1},$

(A3) $w_t + p_t(1-\alpha)\theta(Q_t)H_t^{\beta}K_t^{\alpha}(Q_{t+1} - \gamma Q_t)^{-\alpha} = 0,$

(A4) $p_t\{\theta'(Q_t)H_t^{\beta}K_t^{\alpha}(Q_{t+1} - \gamma Q_t)^{1-\alpha} + \gamma(1-\alpha)\theta(Q_t)H_t^{\beta}K_t^{\alpha}(Q_{t+1} - \gamma Q_t)^{-\alpha}\} = w_{t-1}$

The first two conditions are well-known and state that in maximum marginal profits of investing into knowledge or capital stocks correspond to marginal costs. Production in period *t* generates emissions which negatively affects outputs in period *t+1*. If these impacts are internalized, then w_t is the price, producers at the end of period *t* have to pay for compensating an marginal increase of the atmospheric carbon in period *t+1.* This is what condition (A3) tells. On the other side, it follows from condition (A4) that the compensation w_{t-1}, producers receive at the beginning of period t fully compensates losses in marginal output because damages are higher (First expression on the left side of (A4)) and less can be emitted (second expression on the left side of (A4). By applying some simple manipulations the last two conditions together yield

$$p_t H_t^{\beta}K_t^{\alpha}(Q_{t+1} - \gamma Q_t)^{-\alpha}[\theta'(Q_t)(Q_{t+1} - \gamma Q_t) - \gamma(1-\alpha)\theta(Q_t)] =$$

---

[5] Note $w_t \leq 0$ follows from the fact that carbon stocks Q_t are a bad. For example it could correspond to a tax that has to be paid by polluters when increasing the stock of atmospheric carbon.

$$p_{t-1}(\alpha-1)\theta(Q_{t-1})H_{t-1}^{\beta}K_{t-1}^{\alpha}(Q_t-\gamma Q_{t-1})^{-\alpha}$$

Note, since efficient programs maximize short-run profits, any efficient program fulfills these conditions. And since $\theta'(Q_t)<0, \theta''(Q_t)<0, \alpha, \beta \in (0,1)$, the function $\theta(Q_t)f(H_t,K_t,e_t)$ is concave. Therefore conditions are both nevessary and sufficient for short-run profit maximization. Note finally: If a feasible program is supported by short-run profit maximizing prices and if in addition the transversality condition is satisfied, then the respective program is efficient.

A2 Optimality conditions

the issue of optimal growth was intensively studied (for an overview, see Stephan, 1995). However, the results observed cannot be applied directly, since this literature is usually based on the assumption that there is free disposal. This means in particular that external effects, neither form production nor consumption, are excluded from consideration, which, however, is an essential feature of our analysis.

Now, suppose that the infinite sequence $\{H_t, K_t, Q_t, t=0,1,2,3,\dots\}$ of carbon, capital and knowledge stocks, respectively, defines an interior solution to the optimality problem. Remember that condition (2.2) implies: $Q_{t+1}-\gamma Q_t = e_t$ Then solving the optimality problem

$$max \sum_{t=0}^{\infty} \delta^{-t} U(c_t),$$

subject to

(2.3) $$\theta(Q_t)f(H_t,K_t,e_t) \geq H_{t+1}-\varepsilon_H H_t + K_{t+1} - \varepsilon_K K_t + c_t, \quad t=0,1,2,3,\dots.$$

gives

(A2.5) $$\delta^{-t}U'(c_t)[\beta\theta(Q_t)H_t^{\beta-1}K_t^{\alpha}(Q_{t+1}-\gamma Q_t)^{1-\alpha}+\varepsilon_H]-\delta^{-(t-1)}U'(c_{t-1})=0,$$

(A2.6) $$\delta^{-t}U'(c_t)[\alpha\theta(Q_t)H_t^{\beta}K_t^{\alpha-1}(Q_{t+1}-\gamma Q_t)^{1-\alpha}+\varepsilon_K]-\delta^{-(t-1)}U'(c_{t-1})=0,$$

(A2.7) $$\delta^{-t}U'(c_t)\theta'(Q_t)H_t^{\beta}K_t^{\alpha}(Q_{t+1}-\gamma Q_t)^{1-\alpha} -$$

$$\delta^{-t}U'(c_t)[\gamma(1-\alpha)\theta(Q_t)H_t^{\beta}K_t^{\alpha}(Q_{t+1}-\gamma Q_t)^{-\alpha}] +$$

$$\delta^{-(t-1)}U'(c_{t-1})[(1-\alpha)\theta(Q_{t-1})H_{t-1}^{\beta}K_{t-1}^{\alpha}(Q_t-\gamma Q_{t-1})^{-\alpha}] = 0.$$

Hence by setting $p_t = \delta^{-t}U'(c_t)$, the Euler conditions imply for any $t \geq 1$

(2.5) $$\beta\theta(Q_t)H_t^{\beta-1}K_t^{\alpha}(Q_{t+1}-\gamma Q_t)^{1-\alpha} + \varepsilon_H = \frac{p_{t-1}}{p_t},$$

(2.6) $$\alpha\theta(Q_t)H_t^{\beta}K_t^{\alpha-1}(Q_{t+1}-\gamma Q_t)^{1-\alpha} + \varepsilon_K = \frac{p_{t-1}}{p_t},$$

(2.7) $$\frac{H_t^{\beta}K_t^{\alpha}(Q_{t+1}-\gamma Q_t)^{-\alpha}[\theta'(Q_t)(Q_{t+1}-\gamma Q_t)-\gamma(1-\alpha)\theta(Q_t)]}{(\alpha-1)\theta(Q_{t-1})H_{t-1}^{\beta}K_{t-1}^{\alpha}(Q_t-\gamma Q_{t-1})^{-\alpha}} = \frac{p_{t-1}}{p_t}.$$

The infinite sequence $\{p_t, t = 0,1,2,3,\dots\}$ can be interpreted as system of present value prices. Hence, conditions (2.5) – (2.7) imply that external effects are fully internalized and profits are maximized myopically (see Stephan, 1995).

Since U is concave, we have for any *t* and any $\hat{c} \neq c_t$

$$\delta^{-t}U(c_t) - \delta^{-t}U'(\hat{c}) \geq \delta^{-t}U'(c_t)(c_t-\hat{c}) = p_t(c_t-\hat{c}).$$

Now, since *U(0) = 0*, if a solution to the optimality problem exists and $\{c_t, t = 0,1,2,3,\dots\}$ denotes the corresponding consumption sequence, we observe for any *T*: $\infty > \sum_{t=0}^{\infty}\delta^{-t}U(c_t) \geq \sum_{t=0}^{T}p_t c_t$, which in turn implies the transversality condition

(2.7) $$\sum_{t=0}^{\infty}p_t c_t < \infty.$$

Summing up, we have established a variant of a well-known result:

Proposition: An infinite sequence $\{H_t, K_t, Q_t, t = 0,1,2,3,\dots\}$ *defines an optimal interior program only, if an infinite price sequence* $\{p_t, t = 0,1,2,3,\dots\}$ *exists such that conditions (2,5), (2.6), and (2.7) are fulfilled.*